Equilibrium Shirking, Access to Credit, and Endogenous TFP Fluctuations

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December 12, 2013

Abstract

This paper develops a model in which idea-rich, cash-poor entrepreneurs undertake risky investment projects that are subject to future stochastic liquidity needs and to the potential for shirking due to moral hazard that reduces the likelihood that the projects will succeed. The model suggests that the strength of the incentive for the entrepreneur to shirk is countercyclical, and that endogenous shirking adds volatility to the economy by increasing the persistence and volatility of TFP. This result is the consequence of a greater variation in the number of projects being successfully completed. This variation is compounded by the effect of shirking incentives on the access to credit. The changes in factor employment play only a minor role for the increase in volatility.

Keywords: Credit-rationing, moral hazard, equilibrium shirking, endogenous aggregate TFP.

JEL Codes: D24, D82, E32, E44, E51.

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1 Introduction

The recent financial crisis has refocused the profession on the potentially important role of the financial sector in aggregate economic activity, and has seen renewed interest in the financial accelerator models of Williamson (1986,1987) and Bernanke and Gertler (1989) in which fluctuations in the net worth of businesses as the economy moves through its business cycle can accentuate the cycle's peaks and troughs, thus adding volatility to overall economic activity. These models are premised on a key tenet of modern corporate finance that grew out of the early work of Diamond (1984) in which the presence of private information in lending gives rise to moral hazard issues that can be mitigated through provisions in financial contracts.

This paper presents a model in which idea-rich, cash-poor firms must borrow funds from lenders who face moral hazard whereby entrepreneurs may shirk *ex post* and reduce the likelihood that their projects will be successful.[1] In the model, contract provisions are optimally chosen that allow some endogenous shirking to take place. In a comparison with a version of the model where shirking is ruled out through incentives via financial contracts, the presence of shirking is seen to add significantly to the volatility of the aggregate economy in response to exogenous productivity shocks, primarily through its endogenous contribution to total factor productivity (TFP), with employment of factors by firms largely unaffected. This effect on output and consumption is mainly due to the impact that shirking has on the likelihood of the successful completion of projects.

This paper relates to two strands of the literature on financial frictions.[2] The first one deals with the importance of liquidity constraints in financing. Liquidity shortages are characterized by Holmstrom and Tirole (1998), hereafter HT, and Atolia, Einarsson, and Marquis (2011), hereafter AEM, as arising from the limited pledgeable income associated with funded projects in the presence of moral hazard. Adverse shocks to firms may result in termination of ongoing projects, which in aggregate could reduce overall economic activity if the provision of private liquidity is curtailed. HT examine conditions in a three-period model under which the government may usefully supplement the supply of liquidity to the economy.[3] AEM examine how moral hazard in lending can induce liquidity shortages during severe economic downturns and thereby exacerbate the economic contraction that ensues.

HT-style incentive constraints in equity contracts are always seen to induce maximum work effort. The consequence is a reduction in lending below the socially

[1] In what follows, we will refer to the producing units as 'firms,' or 'entrepreneurs,' or 'projects' depending on what appears appropriate or more natural in the context.

[2] One branch of this literature deals with liquidity issues of financial institutions represented by the bank runs model of Diamond and Dybvig (1983) and models in which financial fragility serves as a commitment mechanism as in Diamond and Rajan (2001). These models rely on adverse selection associated with investor types, which is not treated in this paper or the aforementioned literature.

[3] Kiyotaki and Moore (2005, 2008) examine insufficient aggregate liquidity arising from liquidity constraints that limit the supply of new equity issues due to the inalienable human capital of entrepreneurs and from limited marketability of some existing assets, thus giving rise to a demand for money. Their goal was to demonstrate the potential role of open market operations that are conducted in the equity market in mitigating aggregate liquidity shortages.

optimal level of funding. In AEM, these incentive constraints are seen to bind only occasionally when adverse aggregate economic shocks are sufficiently large, and credit rationing results. The effects of these financial frictions do not depend on a financial accelerator associated with borrower's net worth, as in much of the recent literature, but rather they rely solely on the unwillingness of lenders to support as many firms experiencing idiosyncratic liquidity shocks, and a greater number of firms have their access to credit cut off.

A shortcoming of the AEM model (and other current models) is the rigid structure of the incentive-compatibility constraint that rules out any shirking in equilibrium. In practice, shirking cannot be completely eliminated, nor is it necessarily desirable to do so. This paper examines the consequences of contracts that allow some degree of shirking to occur in equilibrium. This model also includes reproducible physical capital which was absent in AEM. The presence of capital in the model has two significant effects on the model's results. It allows households to smooth consumption through investment, while introducing greater volatility in output due to the persistence effects of fluctuations in reproducible capital on output.[4] The benefit to entrepreneurs from shirking varies across firms and becomes known to the lender after initial funding of the projects. Firms found to be subject to greater moral hazard end up shirking if their projects are taken to completion. Despite shirking, an entrepreneur may receive additional funding for his project, if the project's unanticipated liquidity needs are sufficiently small. That is, while shirking dims the success rate of the project, it does not necessarily make the alternative of no additional funding and zero return with certainty the dominant outcome.

We note that equilibrium shirking qualitatively alters the incidence of moral hazard affecting the firms' access to credit. For example, in the AEM model, moral hazard constraints bind occasionally and directly affect only the initial financing when they bind. In this paper, moral hazard constraints always bind. Moreover, besides affecting the initial financing, they also affect access to additional funding through changes in the shirking status of firms, which represents a new, intensive margin of adjustment.[5] In the AEM model, access to additional funding depends solely on macroeconomic conditions which affects the overall rate of successful completion of projects only on the extensive margin.

The second strand of related literature asks how financial frictions may induce amplification of TFP with respect to exogenous productivity shocks, thus increasing the persistence and volatility of TFP endogenously. Kiyotaki (1998) presents a simplified version of the Kiyotaki and Moore (1997) model in which collateral is a proportion of the future returns from present investment. A temporary shock causes a disproportionately larger reduction in net worth of productive agents due to their existing indebtedness arising from past productive investment. This effect is magnified through the financial accelerator mechanism. There is one similarity between our

[4]See Atolia, Gibson and Marquis (2013) for an analysis of the output and welfare effects in the AEM model with capital added.

[5]Jermann and Quadrini (2012) and Khan and Thomas (2011) offer models with shocks originating in the financial sector as alternative explanations for the role that financial markets play in exacerbating economic downturns.

and Kiyotaki's (1998) simplified model: in both cases the source of incentivization is through apportionment of future returns from present investment. In Kiyotaki (1998), a part has to be credibly committed as collateral for the investment to be financed whereas in our model, a part has to be credibly shared with the entrepreneur (through his stake) for the project to be successfully completed without shirking. In Kiyotaki and Moore (1997) land, a real asset, acts as collateral and net worth fluctuations interact with asset price fluctuations to result in further amplification. Since, capital is rented every period in our model, asset price fluctuations *a la* Kiyotaki and Moore (1997) do not play any role.

Chen and Song (2013) produce a sectoral model in which an exogenous subset of firms is financially constrained. Aggregate productivity shocks lead to a reallocation of factors between the two types of firms due in part to the financial friction, thereby amplifying the effect of productivity shocks on TFP. Khan and Thomas (2011) also present a disaggregated model with a subset of firms financially constrained, which is coupled with irreversible capital decisions to induce large movements in TFP due to shocks originating in the financial sector.

In our model, there are no exogenous shocks that are purely financial in origin; however, the financial sector is seen to contribute significantly to aggregate fluctuations. There is an endogenous response of firms' access to credit that is attributable to two factors: (i) idiosyncratic liquidity shocks, which are modeled as unexpected costs needed to complete partially funded projects, and (ii) firm-specific incentive-compatibility constraints, that are determined by the firm's position in the distribution of private benefits received from shirking. When shirking occurs, it reduces the likelihood of the risky projects succeeding with a positive payoff and also diminishes the expected surplus from successful projects. Firms with a combination of high liquidity needs and tight incentive constraints are cut off from second-period funding required to take their projects to completion. These factors combine to exacerbate economic fluctuations through investment channels. In response to productivity shocks, equilibrium shirking is thus seen to increase the volatility of consumption, output, and labor income by increasing the persistence and volatility of TFP, without having much of an effect on the employment of factors by firms.

2 The Model

The principal focus of the model is on the aggregate consequences of moral hazard when shirking may occur for some firms that are subject to idiosyncratic liquidity shocks. To ensure perfect risk-sharing within the representative household setting, the following assumptions are made. There is a continuum of households, each of which consists of an investor and continuums of workers and entrepreneurs. Each entrepreneur owns an investment project that requires labor services and rented capital that is supplied from outside the household, the funding for which requires external finance. The workers within the household and the household capital stock are supplied to entrepreneurs of other households in exchange for labor and capital income. The investor manages the household's assets, which include capital, equity shares in the projects of other households, and a liquid real asset called 'money.'[6] All entrepreneurs' projects are *ex ante* identical and are traded in a single equity market where they receive identical share prices. A household is able to completely diversify the idiosyncratic risk by taking equal equity positions in all projects offered by other households' entrepreneurs.

In this economy, the household provides neither investment funding nor labor services to its own projects. At the beginning of the period, the members of the household separate, perform their assigned tasks, and then reunite at the end of the period, when they pool their resources and consume together. This structure of the representative household ensures labor and equity markets in which moral hazard issues may arise.

2.1 Project Implementation and Financing

At the beginning of each period, two-period risky projects of total measure one are initiated by the entrepreneurs from each household. These projects are indexed by $i \in [0, 1]$. If the project is taken to completion and is successful, its output is determined by the amount of labor employed in the initial period and the amount of capital rented in the first period and deployed in the second period. Let y^i_{t+1} denote the output of a successfully implemented project that was begun in period t. Then,

$$y^i_{t+1}\left(\theta_{t+1}\right) = \theta_{t+1}(n^i_{1,t})^{\alpha}\left(k^i_{t+1}\right)^{\gamma}, \qquad \alpha, \gamma > 0, \quad \alpha + \gamma \leq 1, \tag{1}$$

where $n^i_{1,t}$ is the outside labor employed by the household in period t, k^i_{t+1} is the capital employed, and θ_{t+1} is the realization of the stochastic aggregate productivity parameter at the beginning of time $t + 1$. Revenue from this project is denoted $\hat{R}^i\left(\theta_{t+1}\right)$ and given by:

$$\hat{R}^i\left(\theta_{t+1}\right) \equiv q^i_{t+1}y^i_{t+1}\left(\theta_{t+1}\right) = q^i_{t+1}\theta_{t+1}(n^i_{1t})^{\alpha}\left(k^i_{t+1}\right)^{\gamma}, \tag{2}$$

where q^i_{t+1} is the price of good i produced by entrepreneur i's project. Note that both the project's output and revenue are stochastic.

All projects require one hundred percent external finance. Investors acquire equity shares that represent claims to the expected future revenues that the funded projects may generate. When the investment is made, the investors take into account three factors relating to the project: (i) they may be required to contribute additional funds at the beginning of the second period in order to bring the project to completion, i.e., no new shares are issued for this purpose; (ii) completed projects may be unsuccessful, yielding zero returns; and (iii) the entrepreneurs may shirk and lower the probability that the project will be successful. To incentivize the entrepreneurs, a higher rate of return on the projects is required, thus creating an underinvestment in equilibrium relative to a full-information environment.[7]

[6] Our treatment of 'money' as a real, liquid asset held for its exchange value is similar to Kiyotaki and Moore (2005) who also abstract from fiat money and inflation.

[7] A referee pointed out that this could be thought of as an "external finance premium" that is

In the first period, entrepreneurs issue equity shares to investors and use the proceeds to finance the first-period wage bill and the capital rental expense. Denote external shares in project i issued by the entrepreneur to investors by s^i and normalize total shares per project to one. Let p_t be the share price associated with a project started in period t, then the entrepreneur receives $p_t s_t^i$ in total initial, first-period funding. Then, the first-period financing constraint is given by:

$$p_t s_t^i = w_t n_{1,t}^i + r_t k_{t+1}^i, \tag{3}$$

where w_t and r_t are the wage rate and capital rental rate in period t.

At the beginning of period $t+1$, each project begun in period t experiences the same aggregate productivity shock, and a project-specific liquidity shock that determines the amount of additional funding needed to bring the project to completion. The shock is denoted ρ_{t+1}^i with distribution $F(\rho)$ and density $f(\rho)$ which are known at date t when the initial financing decision is made. An entrepreneur receiving this shock must hire an additional $n_{2,t+1}^i$ outside workers, such that

$$n_{2,t+1}^i = \rho_{t+1}^i. \tag{4}$$

This shock is labeled as a liquidity shock reflecting the constraint that its financing requires external funding with a liquid asset as described below.

If the project is continued, it may succeed and provide a positive payoff or it may fail and yield a zero return. If the project succeeds, given that all of the project's costs are paid up-front, sales revenues represent net profits to be distributed among shareholders, i.e., investors and entrepreneurs. The probability of success is determined by whether the entrepreneur of a project i chooses to shirk. If he chooses not to shirk the project succeeds with probability p_H, if he chooses to shirk, he receives a private benefit of J_{t+1}^i and the project succeeds with probability $p_L < p_H$, thus inducing a dead-weight loss for the economy. Conditional on the liquidity need being financed, the entrepreneur behaves and chooses not to shirk if his incentive-compatibility constraint

$$p_H \left(1 - s_t^i\right) \hat{R}_{t+1}^i \left(\theta_{t+1}\right) \geq p_L \left(1 - s_t^i\right) \hat{R}_{t+1}^i \left(\theta_{t+1}\right) + J_{t+1}^i s_t^i, \tag{5}$$

is not violated. In (5), $J_{t+1}^i s_t^i$, with $J_{t+1}^i > 0$, is the private benefit from shirking which is an increasing function of outside equity shares, s_t^i, reflecting the lower stake that entrepreneurs hold in the project, i.e., as s increases. The left-hand side of (5) is the benefit from not shirking which must exceed the right-hand side, the total benefit from shirking, for the entrepreneur to not shirk.

In a departure from Atolia, Einarsson, and Marquis (2011), the benefit from shirking, J_{t+1}^i, is uncertain in period t when the project is initially funded, however, its distribution $H\left(J_{t+1}^i\right)$ (with density $h\left(J_{t+1}^i\right)$) is known and the same for all projects. The actual realization occurs at the beginning of period $t+1$ when the liquidity financing decision is made. Given that there is heterogeneity in the realized shirking benefit, then in equilibrium, it is optimal for entrepreneurs to shirk if the benefit from

captured by the entrepreneur.

shirking is above a certain threshold. From (5), this threshold value is

$$J_{t+1}^{i*}(\theta_{t+1}) = \Delta p \frac{1-s_t^i}{s_t^i} \hat{R}^i(\theta_{t+1}) \tag{6}$$

with $\Delta p = p_H - p_L$, such that all projects with $J_{t+1}^i > J_{t+1}^{i*}(\theta_{t+1})$ are subject to shirking and for these projects the probability of success falls from p_H to p_L.

Since the initial financing decision is made prior to the realization of J, all projects receive the same first-period funding. The realization of J is known at the beginning of the next period before the liquidity needs are financed, and it can be observed by the investors. Consequently, the financing of second-period liquidity needs can be made contingent on this realization.[8] The timing of decisions and resolution of uncertainty with respect to projects is shown Figure 1.

Investors realize that there will be a need for liquidity financing in the second period for some fraction of the projects in which they purchase equity shares. This second-period financing requires liquid assets that have been set aside in the previous period for this purpose. For each firm, once ρ_{t+1}^i is observed, the investor decides whether to fund the liquidity need. Conditional on being financed, the expected benefit for the investor is identical for all continued projects. Therefore, there exists a threshold value of ρ_{t+1} such that all projects with lower liquidity needs than the threshold are financed.

The threshold value depends on the aggregate productivity shock and whether that specific project would suffer from shirking. Denote the threshold cutoff value for projects with a high probability of success (no shirking) by $\rho_{t+1}^{H*}(\theta_{t+1})$ and with a low probability of success (shirking) by $\rho_{t+1}^{L*}(\theta_{t+1})$. The dependence of the threshold value of the liquidity shock that determines whether financing is forthcoming on the probability of success owes to: (i) the fact that the probability of success affects the expected benefit of liquidity financing, (ii) the assumption that the actual realization of the benefit from shirking (which affects the probability of success) occurs before the decision to finance the liquidity need is made and that this realization can be observed by the investors, and (iii) the assumption that the liquidity shock and the benefit from shirking are independent. The functional dependence of ρ^*'s on θ_{t+1} arises from the fact that the project revenue (conditional on the probability of success), which determines the benefit of liquidity financing, depends on θ_{t+1}. The interaction of the second-period access to liquidity with the incentive to shirk is crucial to the amplification mechanism highlighted in the paper.

2.2 The Household sector

This section describes the decisions of the representative household – excluding the entrepreneurs' decisions. The entrepreneur's problem is treated separately for added

[8] We do not impose any costly state verification, i.e., an entrepreneur's "type" or private benefit that would accrue in the event of shirking is revealed without additional costs to the investors.

emphasis.[9]

The representative household maximizes lifetime utility, with period utility, $U(C, L)$, defined over consumption and leisure where the varieties of consumption goods produced by different projects are perfect substitutes. Recall, the household consists of the investor, the entrepreneurs, and the workers who specialize in different income-earning activities. The workers' supply labor, n_t, that generates labor income. Entrepreneurs' profits from *maturing* projects, Π_t^l, provide another source of income for the household.

The investor manages the household's assets. He chooses k_{t+1} units of capital to carry to the next period which he rents at rate r_t. The rent on capital is paid in period t. Second, he buys s_t^j shares of projects externally operated by other households, where $j \in [0, 1]$. With the number of project shares normalized to 1, the household is entitled to the fraction s_t^j of the gross revenue from sales of the project's output in period $t+1$, provided the project is carried through to completion and is successful. In order that the project be completed, its random liquidity need that is realized at the beginning of period $t+1$ must be financed. Setting aside funds for future liquidity financing is the third investment option for the household. These funds are carried forward in the form of M_{t+1} units of liquid assets, which consist of the economy's composite goods that can be costlessly stored intertemporally, but yield zero net return.

The final decision of the household's investor is to determine which of the ongoing projects that he has initially funded are to receive additional funding in the second period to absorb the liquidity need and enable the projects to be carried to completion. This decision is made after observing the current period aggregate shock, θ_t, and the project-specific liquidity shock, ρ_t^k, and the revelation of the private benefit, J_t^k, that the entrepreneur managing the project would receive if he were to chose to shirk and lower the probability that the project will succeed. The superscript k differs from j used earlier and is used to indicate projects that were begun and financed in the previous period, $t-1$. As discussed earlier, this decision would take the form of a cut-off value of the liquidity shock, ρ_t^{H*} or ρ_t^{L*} depending on the realization of the benefit from shirking, J_t^k, and the consequent high or low probability of success of the project.

Let $m_t^k\left(\rho_t^k\right)$ denote the liquidity need per share that the household must choose whether to fund, given the number of shares s_{t-1}^k that were issued in the previous period. Then, in equilibrium, given the total liquidity need for project k, $m_t^k\left(\rho_t^k\right)$ satisfies:

$$m_t^k\left(\rho_t^k\right) s_{t-1}^k = \rho_t^k w_t. \tag{7}$$

Denoting the household's total income by X_t, it can be expressed as:

[9] Since many of the household's details are similar to that in AEM, the discussion is restricted to features that are different or vital to the development that follows.

$$X_t = \begin{array}{c} w_t n_t + r_t k_{t+1} + \int_0^1 \Pi_t^l dl + \\ \int_0^1 s_{t-1}^k \hat{R}_t^k(\theta_t) \left[p_H I_{[\rho_t^k \leq \rho_t^{H*}]} I_{[J_t^k \leq J_t^*(\theta_t)]} + p_L I_{[\rho_t^k \leq \rho_t^{L*}]} I_{[J_t^k > J_t^*(\theta_t)]} \right] dk \end{array}, \tag{8}$$

where I denotes the indicator function that is one when the condition in its subscript is true and zero otherwise.

As all goods are perfect substitutes,

$$q_t^k = q_t = Q_t = 1, \tag{9}$$

for all varieties k that are produced in equilibrium as the composite good is taken to be the numeraire.

Thus, the household's budget constraint is

$$\begin{aligned} C_t + \int_0^1 p_t s_t^j dj + \int_0^1 m_t^k(\rho_t^k) s_{t-1}^k \left[I_{[\rho_t^k \leq \rho_t^{H*}]} I_{[J_t^k \leq J_t^*(\theta_t)]} + I_{[\rho_t^k \leq \rho_t^{L*}]} I_{[J_t^k > J_t^*(\theta_t)]} \right] dk \\ + M_{t+1} + \left[K_{t+1} - (1-\delta) K_t \right] \leq M_t + X_t, \end{aligned} \tag{10}$$

where the right-side has the total funds available to the household: the liquidity and the income described in (8). The left-hand side is the use of those funds: consumption, the purchase of shares in new projects, funds needed to meet the liquidity needs of existing projects, provision for the liquidity needs for the next period, and the household's gross investment. In addition to the budget constraint, the ability of the household to meet the current liquidity needs is constrained by the liquidity carried over from the previous period, implying the following inequality:

$$\int_0^1 m_t^k(\rho_t^k) s_{t-1}^k \left[I_{[\rho_t^k \leq \rho_t^{H*}]} I_{[J_t^k \leq J_t^*(\theta_t)]} + I_{[\rho_t^k \leq \rho_t^{L*}]} I_{[J_t^k > J_t^*(\theta_t)]} \right] dk \leq M_t. \tag{11}$$

At the beginning of period 0, the household takes as given its initial asset holdings that include shares in its own projects $(M_0, K_0, s_{-1}^k, s_{-1}^l)$ and solves the following problem:

$$\max_{\substack{\{C_t, L_t, n_t, M_{t+1}, K_{t+1}, \\ s_t^i, s_t^j, \rho_t^{H*}, \rho_t^{L*}\}}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{12}$$

subject to

$$n_t + L_t \leq 1, \tag{13}$$

and $(10-11)$. For the economy as a whole $n_{1,t-1}$ is also given.

2.3 Entrepreneur's Problem

Recall that the project under management by the entrepreneur is subject to moral hazard. The probability of success of the project depends on the effort of the entrepreneur. If the entrepreneur exerts effort, the probability of success is p_H, and if he shirks, the probability falls to $p_L < p_H$. Shirking provides an exogenous benefit to the entrepreneur and investors are aware of this possibility. Being perfectly competitive, the investors provide funding to the point where their expected return is equalized to the return from alternative investment options on a risk-adjusted basis.[10] The entrepreneur collects entire surplus expected to be realized from the investment as profits. These profits represent the agency rents that arise due to the presence of the moral hazard.

We assume that the entrepreneur maximizes his expected profits subject to his incentive compatibility constraint and his first-period funding needs. He is the residual claimant to the fraction $(1 - s_t^i)$ of period $t+1$ gross revenues that are realized if the project succeeds. Ex-post heterogeneity in the benefit from shirking across entrepreneurs allows for the possibility of equilibrium shirking. The incentive-compatiblity constraint in equation (5) defines the threshold value (J^*) of the shirking benefit J (see eq. (6)). For $J > J^*$ the entrepreneur shirks.

The entrepreneur's profits, therefore, are $(1 - s_t^i)\, \hat{R}_{t+1}^i$ with probability $p_H F\left(\rho_{t+1}^{H*}\right)$ if he does not shirk and $p_L F\left(\rho_{t+1}^{L*}\right)$ otherwise. Recall, ρ_{t+1}^{H*} $\left(\rho_{t+1}^{L*}\right)$is the maximum liquidity need that is financed by the investor when the probability of success of the project is high (low). Thus, the entrepreneur's objective becomes

$$\max_{\{s_t^i, n_{1,t}^i, k_{t+1}^i\}} \left\{ \begin{array}{c} (1 - s_t^i)\, p_H E_{t,\theta}\left[\beta \frac{U_{c,t+1}}{U_{c,t}} \hat{R}^i(\theta_{t+1})\, F\left(\rho_{t+1}^{H*}\right) | J_{t+1} \leq J^*(\theta_{t+1})\right] + \\ (1 - s_t^i)\, p_L E_{t,\theta}\left[\beta \frac{U_{c,t+1}}{U_{c,t}} \hat{R}^i(\theta_{t+1})\, F\left(\rho_{t+1}^{L*}\right) | J_{t+1} > J^*(\theta_{t+1})\right] + \\ s_t E_{t,\theta}\left[\beta \frac{U_{c,t+1}}{U_{c,t}} \int_{[J_t > J_t^*(\theta_t)]} J dH(J)\right] \end{array} \right\} \quad (14)$$

where the profits are discounted back to time t using the household's stochastic discount factor and $E_{t,\theta}$ denotes the expectation over θ_{t+1} conditional on information at date t. Also, recall that $H(.)$ is the distribution of J. Equation (14) makes use of the fact that the liquidity shock and the benefit from shirking are independent. The maximization of (14) is subject to the first-period financing constraint, equation (3).

3 Solving the Model

The solution procedure begins with the household's problem followed by that of the entrepreneur. Of interest is the extent to which the measure of projects subject to moral hazard (due to the incentive compatibility constraint for the entrepreneurs) fluctuates over the business cycle. Moreover, to illustrate the model's dynamic properties, attention will focus on the effect of a tightening of the incentive compatibility constraint during a significant adverse shock and how it negatively impacts access to credit for new projects and the funding of liquidity needs for continuing projects, thereby exacerbating economic downturns.

[10] In our set up, as capital rental is paid in advance, capital, the alternative option, is riskless, and the risk-premium for the projects is very small relative to the overall return on the project.

3.1 Solution to the Household's Problem

The first-order conditions for the household's problem yield the familiar Euler equation for the household's labor-leisure choice:

$$w_t U_{C_t} = U_{L_t} \tag{15a}$$

The consumption-savings decision of the household, where savings takes the form of gross investment in capital, is altered only slightly from its familiar form due to the payment in advance that is required by the two-period nature of the projects. In this case, the interest is earned in the current period of the investment and the Euler equation becomes:

$$(1 - r_t)\, U_{C_t} = \beta\,(1 - \delta)\, E_{t,\theta}\left[U_{C_{t+1}}\right] \tag{15b}$$

The optimality conditions for the choice of liquidity (M_{t+1}), levels of investment in projects (s_t^j), and the decision to finance the liquidity needs of the previous-period projects (ρ_t^{H*} and ρ_t^{L*}) are:

$$\begin{aligned}
U_{C_t} &= \beta E_{t,\theta}\left[U_{C_{t+1}}\left\{\frac{p_H \hat{R}_{t+1}\left(\theta_{t+1}\right)}{m_{t+1}\left(\rho_{t+1}^{H*}\right)}\right\}\right], && \text{(15c)}\\
U_{C_t} &= \beta E_{t,\theta}\left[U_{C_{t+1}}\left\{g_{t+1}^{H} H\left(J^*\left(\theta_{t+1}\right)\right) + g_{t+1}^{L}\left[1 - H\left(J^*\left(\theta_{t+1}\right)\right)\right]\right\}\right], && \text{(15d)}\\
U_{C_t} + \lambda_t &= U_{C_t}\frac{p_x \hat{R}_t\left(\theta_t\right)}{m_t\left(\rho_t^{x*}\right)}, \qquad x = H, L, && \text{(15e-15f)}
\end{aligned}$$

where λ_t is the Lagrange multiplier on the liquidity constraint and

$$g_{t+1}^{x} \equiv \left\{\frac{p_x \hat{R}_{t+1}(\theta_{t+1}) F\left(\rho_{t+1}^{x*}\right)}{p_t}\right\}\left\{1 - \frac{\bar{m}_{t+1}^{x}\left(\rho_{t+1}^{x*}\right)}{m_{t+1}\left(\rho_{t+1}^{x*}\right)}\right\} \qquad x = H, L, \tag{16}$$

wherein

$$\bar{m}_{t+1}^{x}\left(\rho_{t+1}^{x*}\right) = \int_0^{\rho_{t+1}^{x*}} m_{t+1}(\rho_{t+1})\frac{f(\rho)}{F\left(\rho_{t+1}^{x*}\right)}d\rho, \qquad x = H, L, \tag{17a-17b}$$

are the average liquidity needs, conditional on the need being financed, when the probability of success of the project is high and low respectively.

Equations (15c) and (15d) reflect the optimal consumption-savings decisions where savings takes the form of money and equity shares. The right-hand side of (15c) represents the discounted expected benefit of foregoing a unit of consumption today in exchange for an increase in the stock of money available to meet future liquidity needs. That is, the term in curly braces is the additional revenues per unit of money carried forward. In equation (16), the first term in curly braces is the expected revenues per share divided by the share price when there is no shirking. The second term in curly braces reflects the additional costs of ownership that the average liquidity needs entail. That is, when the average liquidity need is zero, or $\bar{m} = 0$, then this term is one. As shown in equation (15c), these expected returns are valued at next period's marginal utility, weighted by the probability that no shirking will occur, and discounted back. The second term of (15c) in curly brackets has a similar interpretation for the case of shirking.

Equations (15e) and (15f) reflect the marginal decisions on funding the second-period liquidity needs for the cases when shirking does not occur, (15e), and when shirking does occur, (15f). In the discussion, attention is restricted to (15e), the first-order condition for ρ_t^{H*}, which on simplification yields:

$$\rho_t^{H*}(\theta_t) = \frac{1}{1+\frac{\lambda_t}{U_{C_t}}} \frac{p_H s_{t-1}^k \hat{R}_t^k(\theta_t)}{w_t}. \tag{18}$$

This condition on financing the liquidity need is very intuitive. Should liquidity be in abundant supply, λ_t would be zero, giving:

$$\rho_t^{H*}(\theta_t)\, w_t = p_H s_{t-1}^k \hat{R}_t^k(\theta_t), \tag{19}$$

where the left-hand side is the liquidity need of the marginal project with high probability of success and the right-hand side is the expected revenue accruing to the investor, conditional on the liquidity need being financed. The liquidity need of a project will be financed up to this amount because the past investment decision is not relevant for liquidity financing. In addition, since the investor is diversified over a large number of identical projects, he is risk-neutral with respect to the liquidity funding of any single project. When liquidity is limited, which turns out to be the case in the model solutions, with and without shirking, λ_t is positive and the amount of liquidity supplied to projects is reduced accordingly, as indicated in equation (18).

3.2 Solution to the Entrepreneur's Problem

The entrepreneur's objective in (14) is strictly increasing in $\hat{R}_{t+1}^i(\theta_{t+1})$, the project revenues in the case of successful completion. Given that production costs are paid in advance, the objective is strictly increasing in $n_{1,t}^{\alpha} k_{t+1}^{\gamma}$ irrespective of the future aggregate shock. Thus, it is worthwhile to simplify the problem by first maximizing $n_{1,t}^{\alpha} k_{t+1}^{\gamma}$ subject to the first-period financing constraint. This gives

$$n_{1,t} = \frac{\alpha}{\alpha+\gamma}\frac{p_t}{w_t} s_t, \tag{20a}$$

$$k_{t+1} = \frac{\gamma}{\alpha+\gamma}\frac{p_t}{r_t} s_t. \tag{20b}$$

In what follows, it is assumed that the liquidity shock is uniformly distributed over $[0, \bar{\rho}]$ and the benefit from shirking J is uniformly distributed over $\left[0, \bar{J}\right]$ so that:

$$F(\rho) = \frac{\rho}{\bar{\rho}}, \qquad 0 \leq \rho \leq \bar{\rho}, \tag{21a}$$

$$H(J) = \frac{J}{\bar{J}}, \qquad 0 \leq J \leq \bar{J}. \tag{21b}$$

The first-order condition for this problem, on simplification, yields

$$\begin{aligned}
&[2(\alpha+\gamma+1)(1-s_t)-1]\left(\frac{p_L^2}{p_H^2}\right)E_{t,\theta}\left[\beta\frac{U_{c,t+1}}{U_{c,t}}\theta_{t+1}F\left(\rho_{t+1}^{H*}\right)\right]+ \\
&\left\{\begin{array}{c}[[2+3(\alpha+\gamma)](1-s_t)-2]\left(1-\frac{p_L^2}{p_H^2}\right)- \\ \left[\frac{3}{2}(\alpha+\gamma)(1-s_t)-1\right]\Delta p\frac{p_L}{p_H^2}\end{array}\right\}E_{t,\theta}\left[\beta\frac{U_{c,t+1}}{U_{c,t}}\theta_{t+1}F\left(\rho_{t+1}^{H*}\right)H\left(J^*\left(\theta_{t+1}\right)\right)\right]+ \\
&\frac{2+\alpha+\gamma}{2}\frac{s_t\bar{J}}{n_{1,t}^{\alpha}k_{t+1}^{\gamma}}\frac{p_L}{p_H^2}E_{t,\theta}\left[\beta\frac{U_{c,t+1}}{U_{c,t}}F\left(\rho_{t+1}^{H*}\right)\right]=0
\end{aligned} \tag{22}$$

3.3 Imposing the Equilibrium

The only goods that are produced in equilibrium are from the projects that received a sufficiently low liquidity shock, i.e., for the projects with $\rho_t^i \leq \rho_t^{H*}$ or $\rho_t^i \leq \rho_t^{L*}$ depending on whether the entrepreneur shirks. All projects are *ex ante* identical and all goods enter symmetrically into the utility function. Thus, for each project with $\rho_t^i \leq \rho_t^{H*}$ and $J \leq J^*$ or $\rho_t^i \leq \rho_t^{L*}$ and $J > J^*$, the equilibrium conditions become:

$$s_t^i = s_t, \tag{23a}$$

$$y_t^i = y_t = \theta_t(n_{1,t-1})^{\alpha}(k_t)^{\gamma}, \tag{23b}$$

$$q_t^i = q_t = Q_t = 1, \tag{23c}$$

$$\hat{R}_t^i = \hat{R}_t = y_t \tag{23d}$$

with labor market equilibrium given by:

$$n_{1,t} + \bar{n}_{2,t}^H\left(\rho_t^{H*}\right)F\left(\rho_t^{H*}\right)H\left(J^*\left(\theta_t\right)\right) + \bar{n}_{2,t}^L\left(\rho_t^{L*}\right)F\left(\rho_t^{L*}\right)\left[1-H\left(J^*\left(\theta_t\right)\right)\right] = n_t \tag{24}$$

where

$$\bar{n}_{2,t}^x\left(\rho_t^{i*}\right) = \int_0^{\rho_t^{x*}} \rho\frac{f(\rho)}{F\left(\rho_t^{x*}\right)}d\rho, \qquad x = H, L, \tag{25}$$

are the average additional labor requirements, conditional on the liquidity need being financed, when the probability of success of the project is high and low respectively. Furthermore, the household's time constraint must be satisfied.

$$n_t + L_t = 1. \tag{26}$$

The clearing of the market for the aggregate good requires

$$C_t + M_{t+1} - M_t + K_{t+1} - (1-\delta) K_t = Y_t, \tag{27}$$

where

$$Y_t = y_t(\theta_t)\left\{p_H F\left(\rho_t^{H*}\right) H\left(J^*(\theta_t)\right) + p_L F\left(\rho_t^{L*}\right)\left[1 - H\left(J^*(\theta_t)\right)\right]\right\}, \tag{28}$$

is the output of the aggregate good.

The equilibrium demand for liquidity cannot exceed the supply so that

$$s_{t-1}\left[\begin{array}{c} H\left(J^*(\theta_t)\right)\int_0^{\rho_t^{H*}} m_t(\rho) f(\rho)\, d\rho + \\ \left[1 - H\left(J^*(\theta_t)\right)\right]\int_0^{\rho_t^{L*}} m_t(\rho) f(\rho)\, d\rho \end{array}\right] \le M_t \tag{29}$$

The equations for $(15a - 15f)$, $(20a - 20b)$, (22), $(23b - 23d)$, (24), and $(26 - 29)$ contain the following endogenous variables: s_t, p_t, y_t, $n_{1,t}$, ρ_t^{H*}, ρ_t^{L*}, q_t, $\hat{R}_t$, w_t, r_t, L_t, n_t, K_{t+1}, C_t, M_{t+1}, Y_t, and λ_t. The model thus consists of 17 variables and 17 equations.

4 Calibrating the Model

The functional forms are first specified, followed by the calibration of the model to the data. As the results section assesses the performance of the "Shirking model" of this paper by comparing it to that of the "No Shirking model," we calibrate both variants of the model. For the details of the No Shirking model, the reader is referred to AEM and Atolia, Gibson, and Marquis (2013).

4.1 Functional Forms etc.

The utility function is assumed to be log-linear:

$$U(C, L) = \ln C + \eta \ln L, \ \ \eta > 0 \tag{30}$$

The aggregate productivity shock follows an autoregressive process:

$$\ln \theta_t = \psi_\theta \ln \theta_{t-1} + \varepsilon_t, \tag{31}$$

with serial correlation ψ_θ where the innovation to aggregate productivity, ε_t, is assumed to be normally distributed with mean zero and a standard deviation of σ_ε, but truncated at some lower bound, $\varepsilon_t \ge \varepsilon_L$. Hence, in the non-stochastic steady state $\theta_{ss} = 1$. The truncation of ε under a continuous distribution is necessary to create a lower bound needed for the no shirking version of the model. See AEM for further details.

Given the distribution of the liquidity shock in $(21a)$ and the equilibrium liquidity funding condition (7), equations (25) and (17) can be written as:

$$\begin{aligned}
\bar{n}^x_{2,ss}(\rho^{x*}_{ss}) &= \frac{\rho^{x*}_{ss}}{2}, \qquad x = H, L, && (25_{ss}) \\
\bar{m}^x(\rho^{x*}_{ss}) &= \frac{w_{ss}\rho^{x*}_{ss}}{2s_{ss}} = \frac{m^x(\rho^*_{ss})}{2}, \qquad x = H, L. && (17_{ss})
\end{aligned}$$

Using the functional form of the utility function, the optimality conditions $(15a - 15f)$ can be simplified as follows:

$$\begin{aligned}
\frac{w_{ss}}{C_{ss}} &= \frac{\eta}{L_{ss}}, && (15\mathrm{a}_{ss}) \\
(1 - r_{ss}) &= \beta (1 - \delta), && (15\mathrm{b}_{ss}) \\
1 &= \beta \frac{p_H \hat{R}_{ss}}{m(\rho^{H*}_{ss})} = \beta \frac{s_{ss} p_H \hat{R}_{ss}}{\rho^{H*}_{ss} w_{ss}}, && (15\mathrm{c}_{ss}) \\
p_{ss} &= \beta \hat{R}_{ss} \left[\begin{array}{c} p_H F(\rho^{H*}_{ss}) \left\{ 1 - \frac{\bar{m}^H(\rho^{H*}_{ss})}{m(\rho^{H*})} \right\} H\left(J^*(1)\right) + \\ p_L F(\rho^{L*}_{ss}) \left\{ 1 - \frac{\bar{m}^L(\rho^{L*}_{ss})}{m(\rho^{L*})} \right\} \left[1 - H\left(J^*(1)\right)\right] \end{array} \right], && (15\mathrm{d}_{ss}) \\
\lambda_{ss} &= \frac{1}{C_{ss}} \left[\frac{p_x \hat{R}_{ss}}{m(\rho^{x*}_{ss})} - 1 \right], \qquad x = H, L. && (15\mathrm{e}_{ss}\text{-}15\mathrm{f}_{ss})
\end{aligned}$$

The first-order condition for the entrepreneur's problem also simplifies considerably in the steady state to:

$$\begin{aligned}
&\left[2(\alpha + \gamma + 1)(1 - s_t) - 1\right] \left(\frac{p_L^2}{p_H^2} \right) + \frac{2 + \alpha + \gamma}{2} \frac{s_t \bar{J}}{n^\alpha_{1,t} k^\gamma_{t+1}} \frac{p_L}{p_H^2} + \\
&\left\{ \begin{array}{c} \left[\left[2 + 3(\alpha + \gamma)\right](1 - s_t) - 2\right] \left(1 - \frac{p_L^2}{p_H^2}\right) - \\ \left[\frac{3}{2}(\alpha + \gamma)(1 - s_t) - 1\right] \Delta p \frac{p_L}{p_H^2} \end{array} \right\} H\left(J^*(1)\right) = 0 \qquad (22_{ss})
\end{aligned}$$

4.2 Calibration

Using (4), $(15c_{ss} - 15d_{ss})$ and $(20a)$, one can solve for

$$n_{1,ss} = \frac{\alpha}{\alpha + \gamma} \frac{(\rho^*_{ss})^2}{2\bar{\rho}} \left[H\left(J^*_{ss}\right) + \frac{p_L^2}{p_H^2} \left(1 - H\left(J^*_{ss}\right)\right) \right] = \frac{\alpha}{2\alpha + \gamma} n_{ss}, \qquad (32)$$

where the last equality follows from (24) and (25_{ss}).

The model is calibrated so that in the non-stochastic steady state $n_{ss} = .36$, which approximates a 40-hour workweek and is consistent with the survey data discussed in Juster and Stafford (1991). This implies a value of $\eta = 0.8773$ for the parameter on leisure in the utility function in the case of shirking. In the No Shirking model, the implied value is 0.9906. For an annual calibration, β is set to the usual value of .96. In the production function, α and γ are set to 1/3. The coefficient on capital, γ, is broadly in line with US post-War aggregate data. The labor share parameter, α, is lower than typically assumed in standard (RBC) models. This reflects the fact that the amount of labor devoted to new projects (i.e. initiated in the current period) is only one part of the total hours worked in the economy. The remaining part, $(n-n_1)$, arises from the liquidity shock. Although essential to bring a project to completion, it is 'unproductive' in the sense that the quantity of output is unaffected. This also implies that the marginal products of k and n_1 do not match (are in fact higher than) r and w respectively. Treating entrepreneurs' share $(1-s)$ of net revenue, i.e. $(Y-wn-rk)$, as labor income (payoff for exerting effort), the total share of labor in final output amounts to about 60 percent. Finally, the innovation to aggregate productivity, σ_ε, is set at .0075 in both models. The aggregate shock has a serial correlation of $\psi_\theta = .80$, a value widely assumed in annually calibrated RBC models, broadly equivalent to the quarterly value of 0.95. (See, e.g., Kydland and Prescott (1982).)

To ensure that shirking is costly, p_H is given a relatively high value of .9 and p_L is set to a low value of .4. The liquidity shock distribution parameter $\bar{\rho}$ is set to .7, implying that approximately 90 per cent of the second-period liquidity needs of nonshirking entrepreneurs are financed. For the shirkers, the financing ratio is about 40 per cent. In the No Shirking model, this financing ratio is approximately 85 per cent. The maximum private benefit from shirking, $\bar{J}$, is set to 0.1393, implying that 20 per cent of entrepreneurs are shirking in the nonstochastic steady state of the shirking model. In the No Shirking model, the fixed value of J is set at 0.0846, which implies that the IC constraint in (5) binds roughly one fifth of the time in stochastic simulations.

The preference and technology parameters are listed in Table 1, along with the calibrated steady states of the Shirking and No Shirking models.

5 Results

In this section, the effect of equilibrium shirking on business cycle dynamics is discussed by comparing the outcomes in the Shirking model with the No Shirking model. The second moments and correlation properties of key aggregate variables in simulation exercises are reported. It is seen that shirking adds volatility to output and this volatility is driven by endogenous changes in total factor productivity, arising from changes in the rate of successful completion of the projects, and not so much by changes in the factor usage. Impulse response functions both in the presence and in the absence of equilibrium shirking are then examined which corroborate the findings based on the second moments and the correlations.

5.1 Moral Hazard, Equilibrium Shirking and Business Cycle Volatility

Table 2 presents results for the second moments from simulations of both of the models for the same stochastic process governing the exogenous productivity shock. The presence of shirking results in a significant increase in the volatility of output from 1.98 percent to 3.11 percent. Moreover, the correlation of the technology shock (θ) with output falls from .92 to .85. These facts together imply that the Shirking model has a mechanism that endogenously amplifies and propagates the effect of technology shocks on output, and thereby, to the overall macroeconomy.

To understand this mechanism note that the aggregate output in the Shirking and the No Shirking model is given by

$$Y_t = y_t \left\{ p_H F\left(\rho_t^{H*}\right) H\left(J_t^*\right) + p_L F\left(\rho_t^{L*}\right) \left[1 - H\left(J_t^*\right)\right] \right\}, \tag{33a}$$

$$\tilde{Y}_t = y_t \left\{ p_H F\left(\rho_t^*\right) \right\}, \tag{33b}$$

where ρ^* is the threshold for liquidity financing in the No Shirking model and the term in curly braces is the rate of successful completion of projects. This rate reflects the rate of completion and the probability of success. The former is determined by the financing of the second-period liquidity need. The projects with a liquidity need above the relevant threshold do not get their second-period need financed and are not continued to completion. The latter is determined by whether the entrepreneur shirks. In the Shirking model, variation in the rate of successful completion of projects is also driven by compositional changes arising from changes in the shirking threshold, J^*. Thus, in both models, the statistical properties of aggregate output depend on those of y (the firm-level output conditional on the liquidity need being financed and successful completion of the project) and the liquidity thresholds. However, in the Shirking model, the behavior of Y also depends on that of the shirking threshold J^*.

While we show that fluctuations in J ultimately turn out to dominate the behavior of aggregate output, we begin with the discussion of behavior of the volatility of firm-level output in both models, which shows up one-for-one in volatility of Y (σ_Y) as seen from ($33a - 33b$). As Table 2 shows, y is more volatile in the Shirking model, but only slightly so. The reason is as follows. Access to the first-period financing (ps) is more volatile in the Shirking model (2.50 vs. 1.98) with very similar procyclicality (.98 vs. .97). Thus, firms have better access to funds in good times in the Shirking model. However, most of these extra funds are absorbed by higher payments for wages which are also significantly more volatile (and procyclical) in this model. As a result, there is a very modest increase in factor usage and hence firm-level output (y) in the Shirking model (compared to the No Shirking model).[11]

We now examine the role of access to second-period liquidity financing (or liquidity thresholds). In the No Shirking model, ρ^* is procyclical which increases σ_Y (see ($33b$)). The intuition is clear. Procyclical ρ^* implies that more projects get their second-period liquidity need financed in good times and hence, are successfully completed. To discuss the role of liquidity thresholds in the model with shirking, we first need to

[11] It may be noted that while the availability of first-period funds is strongly procyclical, s actually turns countercyclical in the Shirking model. The reason is the following. While the value of the outside option of the investor fluctuates over the business cycle (see, for example, the volatility of r), this variation is modest relative to the decline in the project surplus during recessions. As a result, the surplus becomes a smaller fraction of the value of the project and hence, a smaller value of $(1 - s)$ allows the entrepreneur to extract the surplus making s countercyclical.

note that (19) implies

$$\rho^{H*}/\rho^{L*} = p_H/p_L \tag{34}$$

so that ρ^{H*} and ρ^{L*} are perfectly correlated. Next, Table 2 shows that ρ^{H*} is countercyclical so that, *for a given* J^*, variation in ρ^{H*} would *reduce* σ_Y in the Shirking model compared to that in the No Shirking model as seen from (33a). Countercyclicality of ρ^{H*} implies that firms that are not subject to binding moral hazard over the entire business cycle have easier access to credit in bad times. This squares with the intuition of the cyclical nature of corporate finance.[12]

The analysis of the common determinants (y and liquidity thresholds) across the two models indicates that while variations in y tend to increase σ_Y in the Shirking model, variations in ρ^{H*} tend to reduce it. However, it is not hard to show that the effect of ρ^{H*} dominates and the overall effect of fluctuations in y and ρ^{H*} is to *reduce* σ_Y. To see that the effect of ρ^{H*} dominates, use (34) in (33a) to obtain

$$Y_t = y_t \left\{ p_H F\left(\rho_t^{H*}\right) \left[H\left(J_t^*\right) + \left(\frac{p_L}{p_H}\right)^2 \left[1 - H\left(J_t^*\right)\right] \right] \right\}. \tag{35}$$

Thus, given the functional form of $F(.)$ in (21a) volatility of ρ^{H*} feeds one-for-one into the volatility of Y, just as in the No Shirking model. The result follows from the fact that there is a considerable increase in the volatility of ρ^{H*} in the Shirking model versus No Shirking model (in comparison to the increase in volatility of y, σ_y) and ρ^{H*} turns significantly countercyclical.

5.1.1 Business Cycle Volatility: Role of Intensive and Extensive Margins

Given that fluctuations in y and liquidity thresholds *ceteris paribus* reduce volatility in the Shirking model, the fluctuation in J^* is the key to higher σ_Y reported in Table 2. These fluctuations provide a second, intensive margin of adjustment which is absent in the No Shirking model.

In the No Shirking model, the adjustment in the rate of successful completion of projects is entirely on the extensive margin. An increase (decrease) in ρ^* increases (decreases) the number of projects that are successfully completed and all projects are similarly affected in terms of access to credit over the business cycle. In the Shirking model, the extensive margin controlled by liquidity thresholds operates in the same manner. But now, there is also a discrete, differential impact on access to credit and the success rate for the marginal projects, viz. projects with J close to J^*, as variation in J^* causes a change in their shirking status. This large, sudden change in access to credit to fund the marginal projects represents the intensive margin of adjustment.

[12] Firms that are always free from moral hazard in the real world (with low J^* in the model) do find it easy to have their liquidity need financed in bad times when there is a 'flight to quality.' On the other hand, firms on the margin of being subject to moral hazard do indeed see their liquidity constraint becoming less binding in good times as the threat to "project success" posed by moral hazard diminishes.

The reasons that the effect of J^* on the intensive margin is strong enough to counteract the effects of other determinants are the following. First, note that J^* is much more volatile than either ρ^{H*} or y. Second, aggregate output is much more sensitive to fluctuations in J^*. To see this latter point, note that as J^* fluctuates over the business cycle, the firms with realizations of J close to the steady-state value of J^* transition from not being subject to shirking to being subject to shirking (and vice versa.) This adjustment on the intensive margin reduces the credit access to the marginal projects by reducing the likelihood of their second-period liquidity need being financed from $F\left(\rho^{H*}\right)$ to $F\left(\rho^{L*}\right)$. In addition, it reduces the likelihood of the project's success from p_H to p_L. Both of these effects on output can be seen in ($33a$). Importantly, these effects positively reinforce each other very strongly as seen from the presence of the $\left(p_L/p_H\right)^2$ term in (35). In fact, from (35), one can easily show that the elasticity of Y with respect to J^* is given by

$$\frac{H\left(J^*_{ss}\right)}{\left[H\left(J^*_{ss}\right)+\left(\frac{p_L}{p_H}\right)^2\left[1-H\left(J^*_{ss}\right)\right]\right]}\left[1-\left(\frac{p_L}{p_H}\right)^2\right], \tag{36}$$

where as $p_H \gg p_L$ implies that both terms are very close to 1. Thus, almost all of the volatility of J^* feeds into σ_Y. In essence, the transition of firms from a state of shirking to no shirking or vice versa due to fluctuations in J^* creates large changes in output by causing large changes in their access to credit and probabilities of their success. This resulting change on the intensive margin is strong enough to result in an overall increase in business cycle volatility, even after overcoming the opposing effect on the extensive margin.

5.2 Financial Frictions and Endogenous Variation in Total Factor Productivity

The reader may have noticed that the factor usage in the economy affects aggregate output, Y, only through y. In particular then, the cyclical variation in the terms in the curly braces in ($33a - 33b$) shows up entirely in that of the total factor productivity (tfp) which is defined as $Y/n_1^{\alpha}k^{\gamma}$. The preceding discussion showed that higher σ_Y in the Shirking model is not driven by an increase in σ_y. Thus, that discussion directly translates into a discussion explaining the higher volatility of tfp (σ_{tfp}) in the Shirking model.

Indeed, as Table 2 shows, σ_{tfp} in the Shirking model is 72 percent higher vis-a-vis the No Shirking model (2.34 vs. 1.36). Thus, shirking and the resultant changes in access to credit due to binding moral hazard constraints significantly amplify the effect of the technology shock on the economy. More strikingly, in the No Shirking model, σ_{tfp} (1.36) is only marginally higher than σ_θ (1.23), the volatility of the exogenous technology shock θ signifying a very weak endogenous amplification mechanism. In contrast, the Shirking model embeds a strong mechanism for endogenous amplification of technology shocks through the credit markets and incentives to shirk as $\sigma_{tfp} \gg \sigma_\theta$ (2.34 vs. 1.23). From, ($33a - 33b$), it is easy to see that the amplification factor, tfp/θ, is the same as the rate of successful completion of the projects. In the No Shirking model, amplification occurs as more projects have their second period liquidity need financed in good times. In the Shirking model, conditional on the state of shirking, projects are less likely to have their liquidity need financed in good times, but this effect is more than offset by the movement of firms/projects from the state of shirking to the state of not shirking. The overall effect is a very strong procyclical movement in the success rate of the projects.

The presence of shirking not only makes tfp more volatile, but it also makes it more persistent. Table 2 shows that, in the model with No Shirking, tfp is more persistent than θ (.87 vs. .80), but it is even more persistent in the Shirking model (.91). The fact that first-order autocorrelation of J^* is .95 (see Table 2) plays an important role in this increase in persistence. The fact that the Shirking model has a mechanism that endogenously raises the persistence of tfp is quite significant. An important criticism of the RBC models is that they require very persistent technology shocks to replicate the observed persistence of the macro variables.

The fact that shirking increases both the volatility and the persistence of tfp (ψ_{tfp}) implies that for the observed values of σ_{tfp} and ψ_{tfp}, the Shirking model would require a less volatile and less persistence technology shock. We undertake a quantitative exercise in this spirit in Table 3. In the exercise, σ_{tfp} is fixed at 1.3 percent. The first column in the table shows the autocorrelation function for tfp in the Shirking model when the standard deviation (σ) of the innovation to θ is 0.0075 as in Table 2, but its first-order autocorrelation (ψ_θ) has been reduced from .8 to .45 to reduce σ_{tfp} to 1.3 percent.

Our objective is to find the values of σ and ψ_θ for which σ_{tfp} and the autocorrelation function for tfp for the No Shirking model are very close to those for the Shirking model. The last column of Table 3 reproduces the benchmark No Shirking model results. Recall in that case, σ_{tfp} is about 1.3 percent. However, note that persistence of tfp is much higher compared to that of the Shirking model in the first column. To get closer to the outcome in the first column, we need to reduce ψ_θ and, to compensate for the resulting decrease in σ_{tfp}, we need to increase σ. This is done in column 3 and as expected, the autocorrelation function gets closer to that in the first column. Column 2 shows that when ψ_θ is reduced to .6 and σ is increased to .0105, σ_{tfp} is 1.3 percent for both models and their autocorrelation functions are also very close, at least for the first few lags. The exercise shows that the Shirking model requires significantly lower σ ($\sim$30 percent lower) and ψ_θ ($\sim$25 percent lower) to give rise to the same level of volatility and persistence in productivity. While exact percentages will vary depending on the actual values of σ_{tfp} and ψ_{tfp}, it is clear that the Shirking model of this paper has significant potential to generate endogenous volatility and persistence in tfp.

5.3 Financial Frictions and the Response to an Adverse Shock

Figure 2 compares the impulse responses of key variables of the two model economies to a sequence of adverse shocks. In the No Shirking model, moral hazard causes the incentive constraint to bind only during severe downturns. Therefore, to contrast the equilibrium shirking case from the no-shirking case in the presence of moral hazard, both economies are subjected to a two-standard deviation adverse productivity shock, beginning in an initial state of economic distress, with below normal productivity ($\theta = .985$).

The aggregate output falls much more sharply in the Shirking model. In accordance with the earlier discussion, one sees from Figure 2 that the fall in output in the economy (especially on impact) is brought about by a proportionately larger endogenous reduction in tfp resulting from a reduction in the rate of successful completion of projects caused by the rising problem of moral hazard. This decline in output is not brought about by a reduction in factor usage, as y - which reflects factor usage - is actually (relatively) higher on impact in the Shirking model. It does turn lower later during transition, but only marginally, resulting in a slightly higher standard deviation as reported in Table 2 when compared to the No Shirking model. The adverse shocks to the economy exacerbate the moral hazard problem and J^* immediately falls (Figure 2) and recovers slowly as the adverse shock dies out. As for ρ^{H*}, it rises (Figure 2) on impact and remains high along the transition in accordance with the earlier discussion. The path of tfp shows that overall the effect of J^* dominates, reducing the rate of successful completion of projects. As a result, the price (p) of shares for the projects and first-period financing fall precipitously. However, as wages also fall steeply, n_1, does not fall as much in the Shirking model, consistent with the earlier assertion of little change in factor usage. Furthermore, the comparison of dynamic responses of Y and J^* in Figure 2 shows, in the end, that the *difference* in the behavior of Y across the two models is essentially governed by that of J^*, as explained previously, and which represents the intensive margin of adjustment in the model.

6 Conclusions

This paper develops a model in which outside equity financing of projects is required along with short-term liquidity needs that arise unexpectedly that require immediate funding for the project to be completed. These projects are subject to the moral hazard associated with entrepreneurial shirking. In this model, equilibrium shirking is present and countercyclical. The operative contracts that permit some degree of shirking to occur are shown to induce greater volatility in output and consumption through increased volatility and persistence of TFP without much effect on employment of factors by firms. This effect on output and consumption is mainly due to the impact that shirking has on the likelihood of the successful completion of projects. We see this mechanism as a promising approach to reducing reliance on volatile and highly persistent exogenous productivity shocks to account for observed business cycle facts.

There are several extensions that this paper has suggested to the authors. How important is entrepreneurial net worth to the incentive for entrepreneurs to shirk and investors to invest in uncertain projects that are subject to this type of moral hazard? In the present set up, the effect of credit tightening is partially mitigated by highly procyclical wages. Reducing wage flexibility, by introducing labor market frictions, can potentially further amplify the effect of credit tightening.

References

Atolia, Manoj, Tor Einarsson, and Milton Marquis. 2011. "Understanding Liquidity Shortages During Severe Economic Downturns," Journal of Economic Dynamics and Control 35: 330-343.

Atolia, Manoj, John Gibson, and Milton Marquis. 2013. "Asymmetry and the Amplitude of Business Cycle Fluctuations: A Quantitative Investigation of the Role of Financial Frictions," Manuscript. Florida State University.

Bernanke, Ben and Mark Gertler. 1989. "Agency Costs, Net Worth, and Business Fluctuations," American Economic Review 79:14-31.

Chen, Kaiji and Zheng Song. 2013. "Financial frictions on capital allocation: A transmission mechanism of TFP fluctuations," Journal of Monetary Economics 60: 683-703.

Diamond, Douglas. 1984. "Financial Intermediation and Delegated Monitoring," Journal of Economic Studies 51:393-414.

Diamond, Douglas and Philip Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity," Journal of Political Economy 91:401-419.

Diamond, Douglas and Raghuram Rajan. 2001. "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking," Journal of Political Economy April.

Holmstrom, Bengt and Jean Tirole. 1998. "Private and Public Supply of Liquidity," Journal of Political Economy 106:1-40.

Jermann, Urban and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks, "American Economic Review, 102(1): 238-271.

Juster, F. Thomas and Frank P. Stafford. 1991. "The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement," Journal of Economic Literature, 29 (June): 471-522.

Khan, Aubhik and Julia K. Thomas. 2011. "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity," NBER Working Paper 17311 (August).

Kiyotaki, Nobuhiro. 1998. "Credit and Business Cycles," Japanese Economic Review 49:18-35.

Kiyotaki, Nobuhiro and John Moore. 1997. "Credit Cycles," Journal of Political Economy 105:211-248.

Kiyotaki, Nobuhiro and John Moore. 2005. "Liquidity and Asset Pricing," International Economic Review 46:317-349.

Kiyotaki, Nobuhiro and John Moore. 2012. "Liquidity, Business Cycles, and Monetary Policy," NBER Working Paper 17934 (March).

Kydland Finn E. and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations," Econometrica 50(4) (November): 1345-1370.

Williamson, Stephen D. 1986. "Costly Monitoring, Financial Intermediation, and Equilibrium Credit Rationing," Journal of Monetary Economics 18:159-179.

Williamson, Stephen D. 1987. "Financial Intermediation, Business Failures, and Real Business Cycles," Journal of Political Economy 95:1196-1216.

Table 1

Preference Parameters

$\beta = 0.96, \;\; \eta = 0.8773 \;\; \text{(shirking)}, \;\; \eta = 0.9906 \; \text{(no shirking)}.$

Maximum Shirking Threshold

$\bar{J} = 0.1393$

Production Parameters

$\alpha = 1/3$

$\gamma = 1/3$

$\delta = 0.05$

$p_H = 0.9, \;\; p_L = 0.4$

$\bar{\rho} = 0.70$

Calibrated Steady State, Shirking

$n = 0.36,$

$\rho^{H*} = 0.6326, \;\; \rho^{H*}/\bar{\rho} = 0.9037$

$\rho^{L*} = 0.2812, \;\; \rho^{L*}/\bar{\rho} = 0.4017$

$J^* = 0.1114, \;\; J^*/\bar{J} = 0.8$

$C = 0.2337, \;\; M = 0.0769, \;\; M/C = 0.33$

$\hat{R} = 0.3742, \;\; Y = 0.2556, \;\; K = 0.4369$

$p = 0.1227, \;\; s = 0.6268$

$w = 0.3204, \;\; r = 0.0880$

Calibrated Steady State, No Shirking

$n = 0.36,$

$\rho^* = 0.5797, \;\; \rho^*/\bar{\rho} = 0.8281$

$J = 0.0846 \;\; J^* = 0.0885,$

$C = 0.2785, \;\; M = 0.1035, \;\; M/C = 0.37$

$\hat{R} = 0.4132, \;\; Y = 0.3079, \;\; K = 0.5879$

$p = 0.1478, \;\; s = 0.7000$

$w = 0.4311, \;\; r = 0.0880$

Table 2

Liquidity Model w/Capital and Equilibrium Shirking

	No Shirking			Shirking		
	$\sigma = 0.0075$			$\sigma = 0.0075$		
Variable	stdev	corr w/Y	autocorr.	stdev	corr w/Y	autcorr.
Y	1.98	1.00	0.93	3.11	1.00	0.94
C	1.56	0.93	0.97	2.68	0.97	0.97
I	9.03	0.76	0.51	11.81	0.72	0.50
K'	2.37	0.80	0.98	3.39	0.84	0.98
θ	1.23	0.92	0.80	1.23	0.85	0.80
y	1.84	0.99	0.91	1.94	0.98	0.92
tfp	1.36	0.95	0.87	2.34	0.97	0.91
ρ^{H*}	0.28	0.46	0.06	1.35	-0.97	0.91
ρ^A	0.56	0.46	0.06	0.61	-0.19	-0.13
w	1.67	0.96	0.97	2.59	0.98	0.97
M'	1.97	0.97	0.87	2.50	0.98	0.91
s	0.17	0.65	0.82	0.53	-0.98	0.91
sp	1.98	0.97	0.87	2.50	0.98	0.91
r	1.51	0.02	0.66	1.97	-0.20	0.75
p	1.87	0.98	0.87	3.03	0.98	0.91
J^*	0.00	-	-	3.29	0.99	0.95
J^*s	-	-	-	2.77	0.99	0.95

Notes: $\rho^A = M/w$

The simulations are based on samples, 3000 periods long.

Table 3

Autocorrelation of TFP

	Shirking	**No shirking**		
σ	0.0075	0.0105	0.0085	0.0075
ψ_θ	0.45	0.60	0.73	0.80
Lag				
1	.71	.72	.82	.87
2	.39	.42	.58	.67
3	.30	.27	.42	.53
4	.24	.17	.32	.40
5	.20	.12	.24	.30
6	.17	.07	.17	.21

Notes: The standard deviation of TFP is 1.3%.

The simulations are based on samples, 3000 periods long.

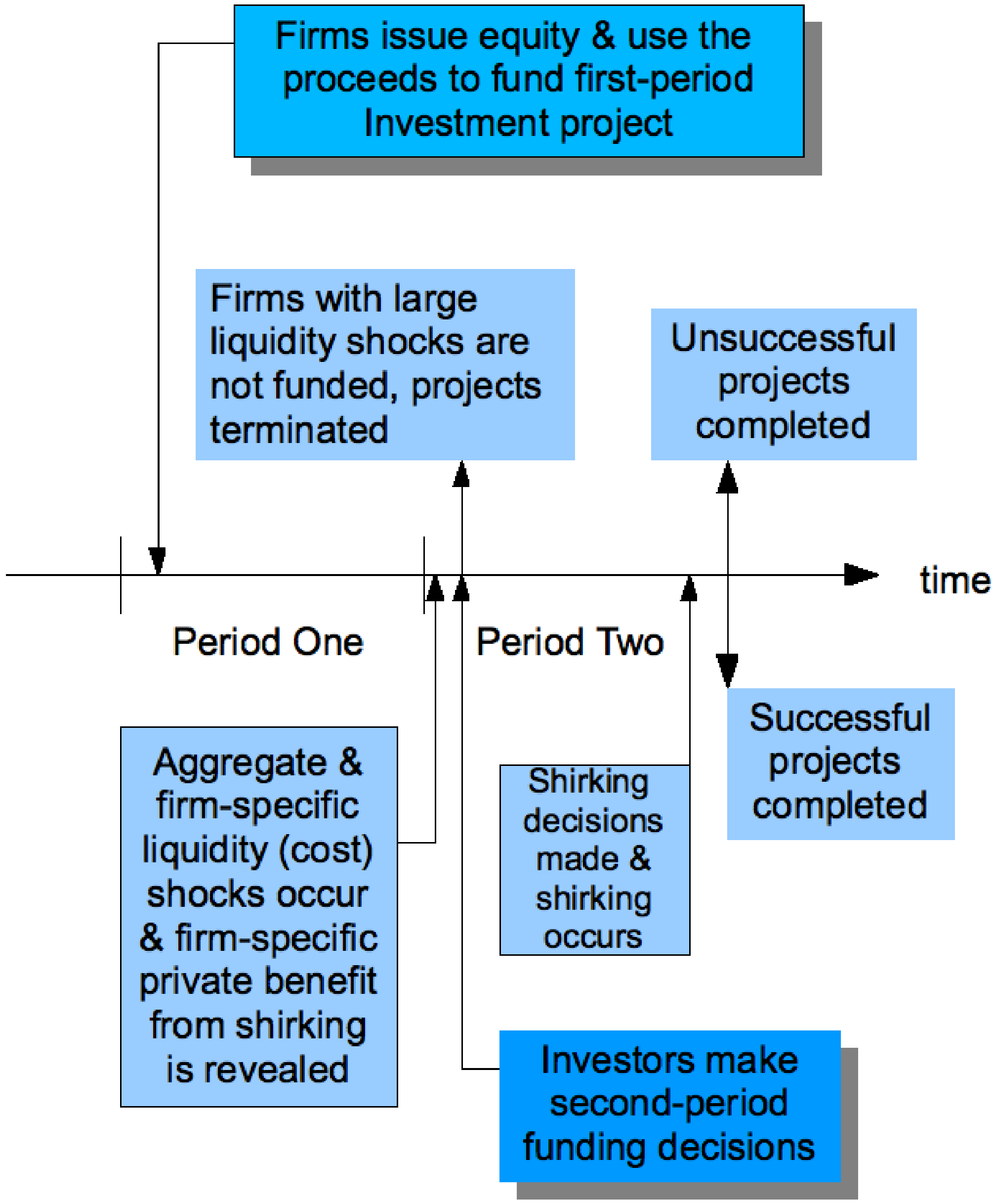

Figure 1. Timing of project financing decisions.

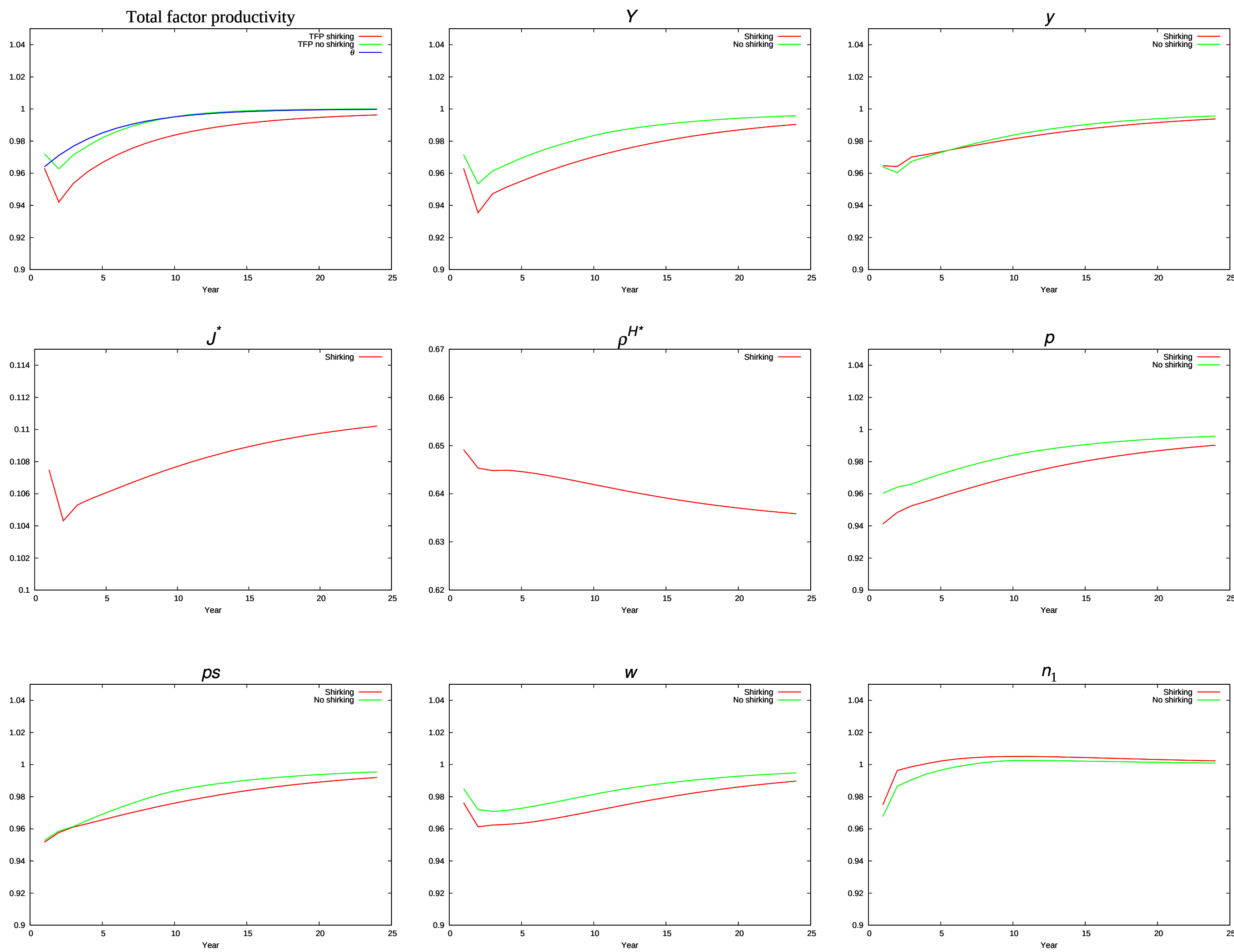

Figure 2. Impulse response functions.